Enhancement and generation of sum squeezing in two-mode light in mixing with coherent light using a beam splitter

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Abstract. Recently, we showed [J. Phys. B: At. Mol. Opt. Phys. **38**, 665 (2005)] that the 'amplitudesquared squeezing', a non-classical feature, can be enhanced in mixing with a coherent light beam using a beam splitter. Here, we show that the sum squeezing in a two-uncorrelated-mode light beam (one mode is in Gaussian state and the other one is in coherent state), which is degenerate limit of amplitude-squared squeezing, may be generated or enhanced in mixing with a two-mode coherent light beam using a beam splitter.

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1 Introduction

The density operator of radiation can be written as

$$
\hat{\rho} = \int d^2 \alpha P(\alpha) \left| \alpha \right\rangle \left\langle \alpha \right| \tag{1}
$$

where $\alpha \equiv \alpha_r + i\alpha_i$ is a complex number, $d^2\alpha \equiv d\alpha_r d\alpha_i$, $|\alpha\rangle$ is the eigenstate of annihilation operator \hat{a} with eigenvalue α (i.e., $\hat{a} | \alpha \rangle = \alpha | \alpha \rangle$), called a coherent state and the carets $($ $)$ denote operators [1]. Weight function $P(\alpha)$ is termed as Glauber-Sudarshan quasiprobability [1] and the radiation is called non-classical when $P(\alpha)$ is not positive-definite or may involve more singular functions than the Dirac delta function [2,3]. Earlier, the non-classical features were studied with only academic interest [4–6] but soon their importance was realized. Squeezing [3–5], antibunching [6], and sub-Poissonian photon statistics [7] were the earliest studied non-classical features. Squeezed states in quantum optics are distinguished by the property that the fluctuations in one of the field quadratures are smaller than those associated with coherent light or a vacuum. Squeezing has received wide attention because of their potential applications not only in reduction of noise level in optical communication [8] and in detection of the extremely weak gravitational radiation [9] but also in the rapidly emerging area of quantum information [10]. The definition of single-mode (normal) squeezing

was generalized by Hong and Mandel [11] who defined the $2N\text{th-order squeezing}$ and by Hillery [12] who defined the amplitude-squared squeezing. Higher-order (twomode) situations were first considered by Hillery [13] in terms of sum and difference squeezing. The property of sum squeezing that it can be converted into normal single-mode squeezing by the sum-frequency generation can be used to detect, *in principle*, the sum squeezing [13] and it has been studied by several authors [14–22]. The concept of sum squeezing first introduced by Hillery for two modes was extended by Kumar and Gupta [18] to the case of three modes. An and Tinh [20] further extended this concept to the most general case of an arbitrary number of modes and derived the relationship between the input general multimode sum squeezing and output normal squeezing. It is important to note that the modes may be photons, phonons, excitons, cooper-pairs, biexcitons, . . . provided that these must obey the Bose-Einstein commutation relations. Hence, the concept of sum squeezing also applies to low-density quasiparticles in condensed matter as well as to the vibrational motion of trapped atoms [20].

Beam splitters [23] are commonly used to mix linearly two optical modes. Recently, we [24] found that *the amplitude-squared squeezing, a non-classical feature, can be enhanced in this simple linear mixing with a classical light beam using a beam splitter*. Since *amplitude-squared squeezing is degenerate limit of sum squeezing* [13], it is natural to investigate whether the sum squeezing can or cannot be enhanced in a similar case, or can even be made to appear when it does not exist in the input light. Also,

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we reported recently [25] an example of enhancement of another non-classical feature in mixing it with a classical light beam using a beam splitter.

In the present paper, we study this problem considering mixing of two-uncorrelated-mode light beam with one mode squeezed Gaussian and the other coherent with a two-mode coherent light beam, using a beam splitter [26]. Parametric amplification [4,27,28] is one such process that generates squeezed Gaussian light beams. Gaussian states of the radiation field are of the greatest importance both theoretically and experimentally. This broad class includes pure states such as coherent and squeezed coherent ones and mixed states such as displaced thermal and squeezed thermal ones [29]. Interest in the non-classical properties of Gaussian states [30] recently renewed by the experimental realization of the teleportation of a one-mode coherent state [31]. Gaussian states have already been utilized in realizations of quantum key distributions [32], teleportation [33], and electromagnetically induced transparency [34]. It is also known that a Gaussian field remains Gaussian by linear transformations that correspond to basic tools in a quantum optics laboratory such as a phase shifter, a beam splitter, and a squeezer [35,36]. An important property of the Gaussian state is that it is completely specified by its mean and its covariance matrix.

2 Sum squeezing in the input light beam and in the output mixed beam

Hillery defined the sum squeezing [13] by considering a more general operator, $\hat{V}_{\phi} = \frac{1}{2} (\hat{a}_1 \hat{a}_2 e^{-i\phi} + \hat{a}_1^{\dagger} \hat{a}_2^{\dagger} e^{i\phi})$, and a negative value of

$$
S_{\phi} \equiv \left\langle (\Delta \hat{V}_{\phi})^2 \right\rangle - \frac{1}{4} \left\langle \hat{N}_{a_1} + \hat{N}_{a_2} + 1 \right\rangle \tag{2}
$$

where $\langle \hat{N}_{a_1} \rangle = \langle \hat{a}_1^{\dagger} \hat{a}_1 \rangle$ and $\langle \hat{N}_{a_2} \rangle = \langle \hat{a}_2^{\dagger} \hat{a}_2 \rangle$ are number operators for modes 1 and 2 with annihilation operators \hat{a}_1 and \hat{a}_2 , respectively. As a quantitative measure of the sum squeezing, we define the squeezing factor for sum squeezing as

$$
S \equiv S_{\phi}/D,\tag{3}
$$

which is bounded by $-1 \leqslant S < 0$ for the case of sum squeezing. Here the denominator, D , is given by

$$
D \equiv \frac{1}{4} \left\langle \hat{N}_{a_1} + \hat{N}_{a_2} + 1 \right\rangle.
$$
 (4)

This definition is, in a way, analogous to the definitions earlier used for quantifying the sub-Poissonian photon statistics [7,37] and the amplitude-squared squeezing [24].

We consider a two-uncorrelated-mode light beam having in one mode (with annihilation operator \hat{a}_1) a beam with Gaussian statistics and normally-ordered characteristic function [38–40] of the form,

$$
\chi_N(\xi) = \text{Tr}[\hat{\rho} e^{\xi \hat{a}_1^{\dagger}} e^{-\xi^* \hat{a}_1}] = \exp[-(A\xi_r^2 + B\xi_i^2)], \quad (5)
$$

and in the other mode (with annihilation operator \hat{a}_2) light in the coherent state $|\alpha_0\rangle$. Even though equation (5) does not represent a very general Gaussian field, rotation and/or displacement operation brings any Gaussian field to this form [38,39]. As the characteristic function $\chi(\xi) =$ $\chi_N(\xi) \exp(-\frac{1}{2}|\xi|^2) \to 0$ for $|\xi| \to \infty$ [39], we have $(A +$ $(\frac{1}{2}) > 0$ and $(B + \frac{1}{2}) > 0$. If $\hat{X}_1 + i\hat{X}_2 = \hat{a}_1$, a state is said to be quadrature squeezed [41] if one of $\langle (\Delta \hat{X}_1)^2 \rangle$ or $\langle (\Delta \hat{X}_2)^2 \rangle$ takes a value $\langle \frac{1}{4} \rangle$. This implies that either B or \overrightarrow{A} is negative because, for light beam with Gaussian statistics, $\langle (\Delta \hat{X}_1)^2 \rangle = \frac{1}{2}B + \frac{1}{4}$ and $\langle (\Delta \hat{X}_2)^2 \rangle = \frac{1}{2}A + \frac{1}{4}$. The uncertainty relation $\langle (\Delta \hat{X}_1)^2 \rangle \langle (\Delta \hat{X}_2)^2 \rangle \ge \frac{1}{16}$ gives another restriction on A and B, viz., $(A+\frac{1}{2})(B+\frac{1}{2}) \geq \frac{1}{4}$ or $(A+B+2AB) \geqslant 0$. For a classical light beam, coefficients A and B are positive. For a non-classical light beam, we have to have $(A+\frac{1}{2})>0$, $(B+\frac{1}{2})>0$ and $(A+B+2AB)\geq$ 0. Thus, for a non-classical light beam, if $B < 0$, i.e., $0 < |B| < \frac{1}{2}$, we have $A > (|B|^{-1} - 2)^{-1} > 0$. Similarly, for $A < 0$, we have $0 < |A| < \frac{1}{2}$ and $B > (|A|^{-1} - 2)^{-1} > 0$.

For mode 1, expectation values of normally-ordered functions of \hat{a}_1 and \hat{a}_1^{\dagger} can be obtained by using the relation,

$$
\text{Tr}[\hat{\rho}_1 \hat{a}_1^{\dagger m} \hat{a}_1^n] = (-1)^n \partial_{\xi}^m \partial_{\xi^*}^n \chi_N(\xi) \big|_{\xi=0}, \qquad (6)
$$

with equation (5) . For the mode 2, it is trivial, as it exists in a coherent state. Expectation values of \hat{V}_{ϕ} and \hat{V}_{ϕ}^2 are, then, seen to be

$$
\langle \hat{V}_{\phi} \rangle = \frac{1}{2} \left[\langle \hat{a}_{1} \hat{a}_{2} \rangle e^{-i\phi} + \langle \hat{a}_{1}^{\dagger} \hat{a}_{2}^{\dagger} \rangle e^{i\phi} \right] = 0, \qquad (7)
$$
\n
$$
\langle \hat{V}_{\phi}^{2} \rangle = \frac{1}{4} \left[\langle \hat{a}_{1}^{2} \hat{a}_{2}^{2} \rangle e^{-2i\phi} + \langle \hat{a}_{1}^{\dagger 2} \hat{a}_{2}^{\dagger 2} \rangle e^{2i\phi} + 2 \langle \hat{a}_{1}^{\dagger} \hat{a}_{1} \hat{a}_{2}^{\dagger} \hat{a}_{2} \rangle + \langle \hat{a}_{1}^{\dagger} \hat{a}_{1} \rangle + \langle \hat{a}_{2}^{\dagger} \hat{a}_{2} \rangle + 1 \right]
$$
\n
$$
= \frac{1}{8} (B - A)(\alpha_{0}^{2} e^{-2i\phi} + \alpha_{0}^{*2} e^{2i\phi}) + \frac{1}{4} (A + B) |\alpha_{0}|^{2}
$$
\n
$$
+ \frac{1}{4} \left[|\alpha_{0}|^{2} + \frac{1}{2} (A + B) + 1 \right]. \qquad (8)
$$

Also,

$$
\langle \hat{N}_{a_1} \rangle = \frac{1}{2}(A+B)
$$
, and $\langle \hat{N}_{a_2} \rangle = |\alpha_0|^2$. (9)

From equations (7) to (9), the values of S_{ϕ} and D can be written as

$$
S_{\phi} = \frac{1}{8}(B-A)(\alpha_0^2 e^{-2i\phi} + \alpha_0^{*2} e^{2i\phi}) + \frac{1}{4}(A+B) |\alpha_0|^2, \tag{10}
$$

and

$$
D = \frac{1}{4} \left\langle \hat{N}_{a_1} + \hat{N}_{a_2} + 1 \right\rangle = \frac{1}{4} \left[|\alpha_0|^2 + \frac{1}{2} (A + B) + 1 \right].
$$
\n(11)

A negative value of S_{ϕ} will be a signature of the nonclassicality, sum squeezing, and this can occur when one of A and B is negative.

Consider a 4-port beam splitter which mixes linearly two two-mode light beams incident at input ports 'a' and

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$$
S_c - S_a = \frac{\left[2\left|B\right|\left(\left|\alpha_0\right| + \left|\beta\right|\right)^2 \left(2\left|\alpha_0\right|^2 - 2 - A + \left|B\right|\right) + 4\left|B\right|\left|\alpha_0\right|^2 \left(4 + A - \left|B\right|\right)\right]}{\left[\left(2 + A - \left|B\right| + 2\left|\alpha_0\right|^2\right)\left(4 + A - \left|B\right| + 2\left(\left|\alpha_0\right| + \left|\beta\right|\right)^2\right)\right]}
$$
\n(19)

Fig. 1. Beam splitter (BS) with two inputs and two outputs having two-mode light beams $(j = 1, 2)$.

'b' and generates two output light beams at ports 'c' and 'd'. The annihilation operators of these eight modes are then related by

$$
\hat{c}_j = t_j \hat{a}_j + ir_j \hat{b}_j, \quad \hat{d}_j = t_j \hat{b}_j + ir_j \hat{a}_j, \tag{12}
$$

where \hat{a}_j , \hat{b}_j , \hat{c}_j , and \hat{d}_j (j = 1, 2) are annihilation operators for the two modes at ports 'a', 'b', 'c' and 'd' and t_j and r_i are real transmission and reflection coefficients [42] for the jth mode, respectively (see Fig. 1). If the Gaussian light under consideration and coherent light in state $|\alpha_0\rangle$ are incident at input port 'a' and light in coherent state $|\alpha, \beta\rangle$ is incident at input port 'b', straightforward calculation gives the values of S_{ϕ} and D at port 'c' as

$$
S_{\phi c} = \frac{1}{8}(B - A)[t_1^2 t_2^2 \alpha_0^{*2} - r_2^2 t_1^2 \beta^{*2} - 2ir_2 t_2 t_1^2 \alpha_0^{*} \beta^{*}]e^{2i\phi} + \frac{1}{8}(B - A)[t_1^2 t_2^2 \alpha_0^2 - r_2^2 t_1^2 \beta^2 + 2ir_2 t_2 t_1^2 \alpha_0 \beta]e^{-2i\phi} + \frac{1}{4}(A + B)[t_1^2 t_2^2 |\alpha_0|^2 + t_1^2 r_2^2 |\beta|^2 + ir_2 t_2 t_1^2 (\alpha_0^{*} \beta - \alpha_0 \beta^{*})],
$$
(13)

and

$$
D_c \equiv \frac{1}{4} \left\langle \hat{N}_{c_1} + \hat{N}_{c_2} + 1 \right\rangle = \frac{1}{4} [t_2^2 | \alpha_0 |^2 + r_1^2 | \alpha |^2 + r_2^2 | \beta |^2 + \frac{1}{2} t_1^2 (A + B) + i r_2 t_2 (\alpha_0^* \beta - \alpha_0 \beta^*) + 1].
$$
 (14)

The values of S_{ϕ} and D at the input port 'a' are given by equations (10) and (11), and henceforth we will denote them as $S_{\phi a}$ and D_a , respectively.

3 Discussions

We may write $\alpha_0 \equiv |\alpha_0| e^{i\theta_0}, \beta \equiv |\beta| e^{i\theta_\beta}$, define $\eta \equiv \phi - \theta_0$ and $\varsigma \equiv \phi - \theta_{\beta}$, and rewrite equations (10), (11), (13) and (14) in the forms,

$$
S_{\phi a} = \frac{1}{4} |\alpha_0|^2 [(B - A)\cos 2\eta + (A + B)],
$$
 (15)

$$
D_a = \frac{1}{4} [|\alpha_0|^2 + \frac{1}{2}(A+B) + 1],
$$
\n
$$
S_{\phi c} = \frac{1}{4} t_1^2 t_2^2 [(B-A)\cos 2\eta + (A+B)] |\alpha_0|^2
$$
\n
$$
+ \frac{1}{4} r_2^2 t_1^2 |\beta|^2 [(A-B)\cos 2\varsigma + (A+B)]
$$
\n
$$
+ \frac{1}{2} r_2 t_2 t_1^2 |\alpha_0| |\beta| [(B-A)\sin(\eta + \varsigma)]
$$
\n
$$
- (A+B)\sin(\eta - \varsigma)],
$$
\n(17)

and

$$
D_c = \frac{1}{4} [t_2^2 |\alpha_0|^2 + r_1^2 |\alpha|^2 + r_2^2 |\beta|^2 + \frac{1}{2} t_1^2 (A + B) - 2r_2 t_2 |\alpha_0| |\beta| \sin(\eta - \varsigma) + 1].
$$
 (18)

We now examine the sum squeezing in output at the port 'c' and show its enhancement or appearance whether the two-uncorrelated-input-modes at port 'a' is sum squeezed or is not. Also, we consider $|\alpha|^2 = 0$, $r_1^2 = t_1^2 = r_2^2 = t_2^2 = \frac{1}{2}$ for simplicity. We observe the following:

Case I: consider the case $A > 0$, $B < 0$, and (η, ς) $(0, \pi/2)$, where equation (15) reveals that $S_{\phi a}$, and therefore $S_a \equiv S_{\phi a}/D_a$, is negative and the input is sum squeezed. Equations (15) to (18), then, give

see equation (19) above.

Here $S_c \equiv S_{\phi c}/D_c$. We can easily visualize cases where the numerator is negative, and hence $S_c < S_a < 0$ and $|S_c| > |S_a|$. Clearly, the sum squeezing has increased in such a case. One simple case can, e.g., be $|\alpha_0| = 0$, for which the numerator can be written as $2B|\beta|^2$ [{-1 – $(A + \frac{1}{2})$ } – $(\frac{1}{2} - |B|)$] which is clearly negative and the denominator as $\left[\frac{3}{2} + A + \left(\frac{1}{2} - |B|\right)\right] \left[\frac{7}{2} + A + \left(\frac{1}{2} - |B|\right) + \right]$ $2|\beta|^2$ which is clearly positive. A similar situation can be discussed with A interchanged with B and η with ς. In these cases, although the input light beams are sum squeezed and the sum squeezing increases in mixing with coherent light;

Case II: we can also show that, even if the input light beam is not sum squeezed, the output beam can become sum squeezed. Some such cases are (i) $A > 0$, $B < 0$ and one of η and ζ equal to $\pi/2$ and the other equal to $3\pi/2$, and (ii) $A < 0$, $B > 0$ and one of η and ζ equal to 0 and the other equal to π . One simple example is $|\alpha_0| = 0$, $r_1^2 = t_1^2 = r_2^2 = t_2^2 = \frac{1}{2}$, for which equations (15) to (18), give $S_a = 0$, and $S_c = |\beta|^2 [A + B + (A - B) \cos 2\varsigma]/(4 +$ $A + B + 2|\beta|^2$. Clearly, $S_c < S_a = 0$ for, e.g., (i) $A < 0$, $B > 0, \varsigma = 0$ and (ii) $A > 0, B < 0, \varsigma = \pi/2$. In these cases, although the input light beam is not sum squeezed, sum squeezing appears in the output.

Fig. 2. Plot of S_a and S_c vs. η with $B = -0.49$, $A = 25$, $|\alpha|^2 = 0$, $|\alpha_0|^2 = 1$, $|\beta|^2 = 25$, $\zeta = \pi/2$ (S_a is represented by '−' and S_b by '+') S_c by $'+$).

These results can be illustrated by plotting, e.g., S_a and S_c for different values of η keeping ζ fixed. In Figure 2, we do this taking $r_1^2 = t_1^2 = r_2^2 = t_2^2 = \frac{1}{2}$, $|\alpha|^2 = 0$, $A = 25$, $B = -0.49, |\alpha_0|^2 = 1, |\beta|^2 = 25, \text{ and } \varsigma = \pi/2.$ This may correspond to an experiment where θ_0 , the phase of coherent input beam at port a' , is varied. We see clearly regions where (i) $S_c < S_a < 0$, i.e., *sum squeezing is enhanced*, and (ii) $S_c < 0$ but $S_a = 0$ where, *although the input beam is not sum squeezed, the output mixed beam becomes sum squeezed*.

It may be recalled that in mixing of an input quadrature squeezed Gaussian light beam with a coherent beam, we showed [24] that the output beam couldn't exhibit amplitude-squared squeezing [12,24] unless the input Gaussian beam is amplitude-squared squeezed. This is *in contrast* with the present case of sum squeezing where *output light beam may become sum squeezed even if the input light beam is not sum squeezed*. It is important to note that amplitude-squared squeezing and sum squeezing is occasionally referred to in the literature as oneand two-mode $SU(1,1)$ squeezing. Since the modes may be photons, phonons, excitons, cooper-pairs, biexcitons, . . . provided that these must obey the Bose-Einstein commutation relations, such an analysis of the sum squeezing can have also application to low-density quasiparticles in condensed matter as well as to the vibrational motion of trapped atoms [20]. The theoretical predictions of this paper can be detected experimentally easily in a simple case (case II) discussed in Section 3. If we consider a Gaussian non-classical beam incident at one input port of the beam splitter and a coherent beam with a different frequency at the other input port, the input light is not sum squeezed but the output light beam shows sum squeezing. Sum squeezing can be detected, in principle, experimentally [13, 18]. It is also to be noted that a similar investigation was done by the present authors for difference squeezing [13], but no such an enhancement occurred.

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